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# Single Spin Asymmetry in Charmonium Production

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**Abstract** We present estimates of Single Spin Asymmetry (SSA) in the electroproduction of  $J/\psi$  taking into account the transverse momentum dependent (TMD) evolution of the gluon Sivers function and using Color Evaporation Model of charmonium production. We estimate SSA for JLab, HERMES, COMPASS and eRHIC energies using recent parameters for the quark Sivers functions which are fitted using an evolution kernel in which the perturbative part is resummed up to next-to-leading logarithms (NLL) accuracy. We find that these SSAs are much smaller as compared to our first estimates obtained using DGLAP evolution but are comparable to our estimates obtained using TMD evolution where we had used approximate analytical solution of the TMD evolution equation for the purpose.

**Keywords** Charmonium, SSA, TMD evolution

## 1 Introduction

Transverse Single Spin Asymmetries (SSA's) arise in the scattering of a transversely polarized nucleon off an unpolarized nucleon (or virtual photon) target when the final observed hadrons have asymmetric distribution in the transverse plane perpendicular to the beam direction depending on the polarization vector of the scattering nucleon. One of the two major theoretical approaches to explain these asymmetries is the Transverse Momentum Dependent (TMD) approach[1; 2] which is based on a pQCD factorization scheme which includes the spin and TMD effects in the collinear factorization scheme. An important Transverse Momentum Dependent Distribution (TMD) is the Sivers function which is related to the density of unpolarized partons in a transversely polarized nucleon.

The number density of partons inside proton with transverse polarization  $\mathbf{S}$ , three momentum  $\mathbf{p}$  and intrinsic transverse momentum  $\mathbf{k}_\perp$  of partons, is expressed in terms of the Sivers function,  $\Delta^N f_{a/p}(x, k_\perp)$ , as

$$\hat{f}_{a/p^\uparrow}(x, \mathbf{k}_\perp) = \hat{f}_{a/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{a/p^\uparrow}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \quad (1)$$

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Heavy quark and quarkonium systems are natural probes to study gluon Sivvers function as the production and the differential distributions of the produced charmonium are directly dependent on the intrinsic transverse momentum distribtuion of the gluon especially at low transverse momentum[3]. We have proposed study of SSA in  $J/\psi$  production as a possible probe of gluon Sivvers function and have estimated SSA in photo production (i.e. low virtuality electro production) of  $J/\psi$  in scattering of electrons off transversely polarized protons, using the color evaporation model (CEM) of charmonium production[4; 5]. Here, we present some preliminary results containing improved estimates of asymmetry taking into account the TMD evolution of the Sivvers function up to next-to-leading logarithm (NLL) order. A more detailed analysis can be found in Ref. [6].

## 2 Transverse Single Spin Asymmetry in $e + p^\uparrow \rightarrow J/\psi + X$

In the process under consideration, at LO, there is contribution only from a single partonic subprocess  $\gamma g \rightarrow c\bar{c}$  and hence it provides a clean probe of gluon Sivvers function. The CEM expression for electro-production of  $J/\psi$  can be generalized by taking into account the transverse momentum dependence of the William Weizsaker (WW) function and gluon distribution function and can be written as

$$\sigma^{e+p^\uparrow \rightarrow e+J/\psi+X} = \int_{4m_c^2}^{4m_D^2} dM_{c\bar{c}}^2 dx_\gamma dx_g d^2\mathbf{k}_{\perp\gamma} d^2\mathbf{k}_{\perp g} f_{g/p^\uparrow}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/e}(x_\gamma, \mathbf{k}_{\perp\gamma}) \frac{d\hat{\sigma}^{\gamma g \rightarrow c\bar{c}}}{dM_{c\bar{c}}^2}$$

where  $f_{\gamma/e}(y, E)$  is the distribution function of the photon in the electron given by William Weizsaker approximation [7]. We assume  $k_\perp$  dependence of pdf's and WW function to be factorized in gaussian form [8; 4].

$$f(x, k_\perp) = f(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle} \quad \langle k_\perp^2 \rangle = 0.25 GeV^2$$

We use the following model for gluon Sivvers function proposed by Anselmino *et al.*[8]

$$\Delta^N f_{g/p^\uparrow}(x, k_\perp) = 2\mathcal{N}_g(x) \sqrt{2} e \frac{k_\perp}{M_1} e^{-k_\perp^2 / M_1^2} f_{g/p}(x) \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle} \cos \phi_{k_\perp}$$

$\mathcal{N}_g(x)$  is the  $x$ -dependent normalization for which we have used  $\mathcal{N}_g(x) = \mathcal{N}_d(x)$ . Since there is no information on gluon Sivvers function, one parameterizes the gluon Sivvers function in terms of quark Sivvers function. The  $x$ -dependent normalization for quarks is [8]

$$\mathcal{N}_f(x) = N_f x^{a_f} (1-x)^{b_f} \frac{(a_f + b_f)^{(a_f + b_f)}}{a_f^{a_f} b_f^{b_f}}$$

where  $a_f, b_f$  and  $N_f$  are best fit parameters. The weighted Sivvers asymmetry integrated over the azimuthal angle of  $J/\psi$ [9] is given by

$$A_N = \frac{\int d\phi_q \int_{4m_c^2}^{4m_D^2} [dM^2] \int d^2\mathbf{k}_{\perp g} \Delta^N f_{g/p^\uparrow}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/e}(x_\gamma, \mathbf{q}_T - \mathbf{k}_{\perp g}) \hat{\sigma}_0 \sin(\phi_q - \phi_S)}{2 \int d\phi_q \int_{4m_c^2}^{4m_D^2} [dM^2] \int d^2\mathbf{k}_{\perp g} f_{g/P}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/e}(x_\gamma, \mathbf{q}_T - \mathbf{k}_{\perp g}) \hat{\sigma}_0} \quad (2)$$

where  $\phi_q$  and  $\phi_S$  are the azimuthal angles of the  $J/\psi$  and proton spin respectively. The weight factor is  $\sin(\phi_q - \phi_S)$  and  $x_{g,\gamma} = \frac{M}{\sqrt{s}} e^{\pm y}$ .

## 3 QCD evolution of TMDs

Early phenomenological fits of Sivvers function were performed either neglecting QCD evolution or applying DGLAP evolution only to the collinear part of TMD parametrization. Recently a TMD factorization formalism has been derived and implemented by Collins *et al.* [1]. TMD evolution describes how the form of distribution changes and also how the width changes in momentum space. A strategy to extract Sivvers function from SIDIS data taking into account the TMD  $Q^2$  evolution was proposed by Anselmino *et al.*[10]. In our earlier work, we estimated SSA in electroproduction of  $J/\psi$  using this

strategy [5]. The energy evolution of a general transverse momentum dependent distribution(TMD)  $F(x, k_\perp, Q)$  is more naturally described in  $b$ -space. The TMDPDF in  $b$ -space evolves according to

$$F(x, b, Q_f) = F(x, b, Q_i) R_{pert}(Q_f, Q_i, b_*) R_{NP}(Q_f, Q_i, b) \quad (3)$$

where  $R_{pert}$  is the perturbative part of the evolution kernel,  $R_{NP}$  is the non-perturbative part and  $b_* = b/\sqrt{1 + (b/b_{\max})^2}$ . The perturbative part is given by

$$R_{pert}(Q_f, Q_i, b) = \exp \left\{ - \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left( A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left( \frac{Q_f^2}{Q_i^2} \right)^{-D(b; Q_i)} \quad (4)$$

where  $\frac{dD}{d \ln \mu} = \Gamma_{\text{cusp}}$ . The non-perturbative exponential part contains a  $Q$ -dependent factor universal to all TMD's ( $g_2$ ), and a factor which gives the gaussian width in  $b$ -space of the particular TMD ( $g_1$ ).

$$R_{NP} = \exp \left\{ -b^2 \left( g_1^{\text{TMD}} + \frac{g_2}{2} \ln \frac{Q_f}{Q_i} \right) \right\}$$

The  $Q^2$ -dependent TMD's in momentum space are obtained by Fourier transforming  $F(x, b, Q_f)$ . The perturbative evolution kernel  $R(Q, Q_0, b)$ , which drives the  $Q^2$ -evolution of TMD's, becomes independent of  $b$  in the limit  $b \rightarrow \infty$  i.e.  $R(Q, Q_0, b) \rightarrow R(Q, Q_0)$ . The  $b$  integration can then be performed analytically and  $Q^2$  dependent PDF's can be obtained. In Ref. [5], we had used this analytical solution of approximated TMD evolution equations given by Anselmino *et al.* [10] to estimate the asymmetry.

Here, we present our improved estimates based on exact solution of evolution equations. Echevarria *et al.*[11] have recently considered solution of TMD evolution equations up to NLL accuracy and have performed a global fit of all the experimental data on the Sivers asymmetry in SIDIS using this formalism. Since the derivative of  $b$ -space Sivers function satisfies the same evolution equation as the unpolarized PDF[12], its evolution is given by

$$\begin{aligned} f_{1T}^{\perp g}(x, b; Q_f) &= \frac{M_p b}{2} T_{g,F}(x, x, Q_i) \exp \left\{ - \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left( A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left( \frac{Q_f^2}{Q_i^2} \right)^{-D(b^*; Q_i)} \\ &\times \exp \left\{ -b^2 \left( g_1^{\text{sivers}} + \frac{g_2}{2} \ln \frac{Q_f}{Q_i} \right) \right\} \end{aligned} \quad (5)$$

Here,  $T_{q,F}(x, x, Q)$  is the twist three Qiu-Sterman quark gluon correlation function which is related to the first  $k_T$  moment of quark Sivers function[13] and can be expressed in terms of the unpolarized collinear PDFs [14; 11].

$$T_{q,F}(x, x, Q) = \mathcal{N}_q(x) f_{q/P}(x, Q) \quad (6)$$

The expansion coefficients with the appropriate gluon anomalous dimensions at NLL are

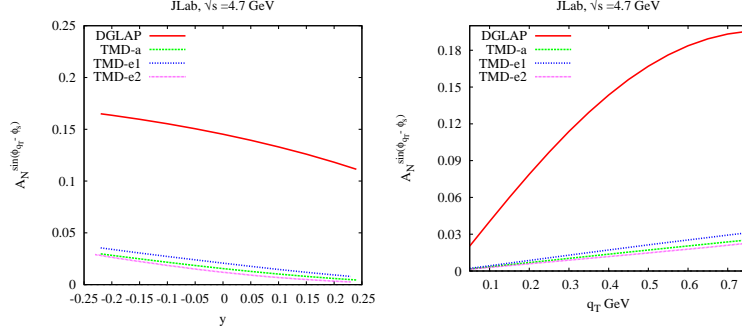
$$A^{(1)} = C_A; A^{(2)} = \frac{1}{2} C_F \left( C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} C_A N_f \right); B^{(1)} = -\frac{1}{2} \left( \frac{11}{3} C_A - \frac{2}{3} N_f \right); D^{(1)} = \frac{C_A}{2} \ln \frac{Q_i^2 b^2}{c^2}$$

Choosing the initial scale  $Q_i = c/b$ , the  $D$  term vanishes at NLL. Taking Fourier transform of Eq. (6), one gets  $f_{1T}^{\perp g}(x, k_\perp; Q_f)$  which is related to Sivers function through

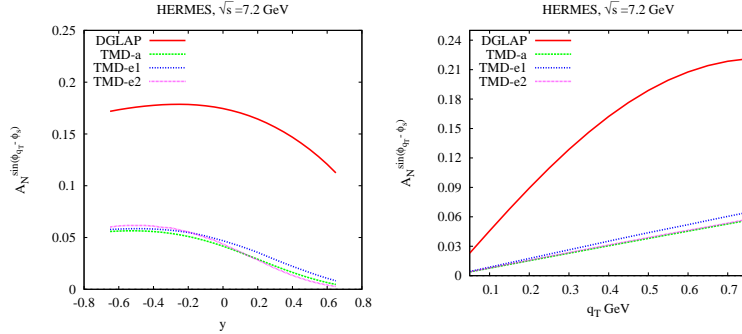
$$\Delta^N f_{g/p^\uparrow}(x_g, \mathbf{k}_{\perp g}, Q) = -2 \frac{k_{\perp g}}{M_p} f_{1T}^{\perp g}(x_g, k_{\perp g}; Q) \cos \phi_{k_\perp} \quad (7)$$

TMD-e1	TMD-a	TMD-e2
$N_u = 0.77, N_d = -1.00$	$N_u = 0.75, N_d = -1.00$	$N_u = 0.106, N_d = -0.163$
$a_u = 0.68, a_d = 1.11$	$a_u = 0.82, a_d = 1.36$	$a_u = 1.051, a_d = 1.552$
$b_u = b_d = 3.1,$	$b_u = b_d = 4.0,$	$b_u = b_d = 4.857,$
$M_1^2 = 0.40 \text{ GeV}^2$	$M_1^2 = 0.34 \text{ GeV}^2$	$\langle k_{\perp}^2 \rangle = 0.282 \text{ GeV}^2$
$\langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2$	$\langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2$	$\langle k_{\perp}^2 \rangle = 0.38 \text{ GeV}^2$
$b_{max} = 0.5 \text{ GeV}^{-1}$	$b_{max} = 0.5 \text{ GeV}^{-1}$	$b_{max} = 1.5 \text{ GeV}^{-1}$
$g_2 = 0.68 \text{ GeV}^2$	$g_2 = 0.68 \text{ GeV}^2$	$g_2 = 0.16 \text{ GeV}^2$

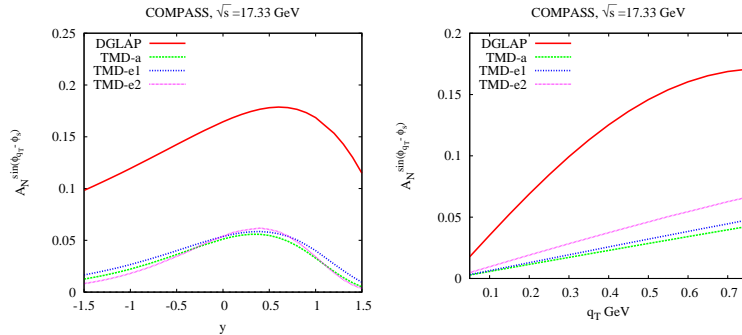
**Table 1** Parameter set for the Siverts function.



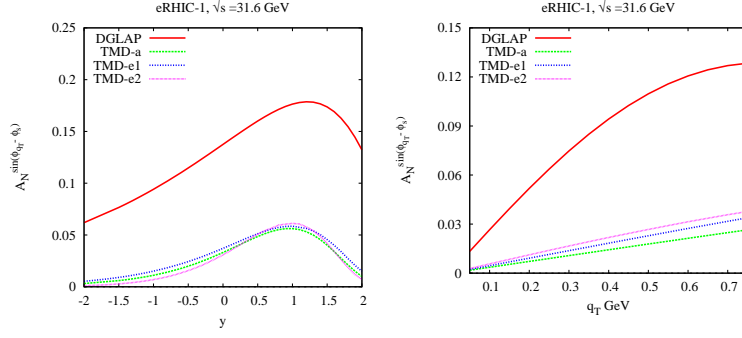
**Fig. 1** The Siverts asymmetry  $A_N^{\sin(\phi_{qT} - \phi_S)}$  for  $e + p^+ \rightarrow e + J/\psi + X$  at JLab energy ( $\sqrt{s} = 4.7 \text{ GeV}$ ), as a function of  $y$  (left panel) and  $q_T$  (right panel). The integration ranges are  $(0 \leq q_T \leq 1) \text{ GeV}$  and  $(-0.25 \leq y \leq 0.25)[6]$ .



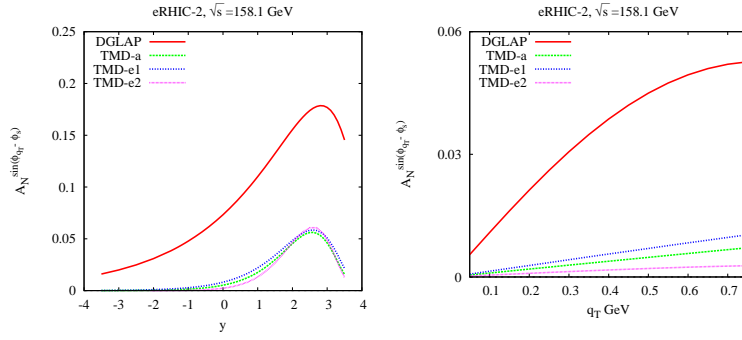
**Fig. 2** HERMES energy ( $\sqrt{s} = 7.2 \text{ GeV}$ ), Asymmetry as a function of  $y$  (left panel) and  $q_T$  (right panel). The integration ranges are  $(0 \leq q_T \leq 1) \text{ GeV}$  and  $(-0.6 \leq y \leq 0.6)[6]$ .



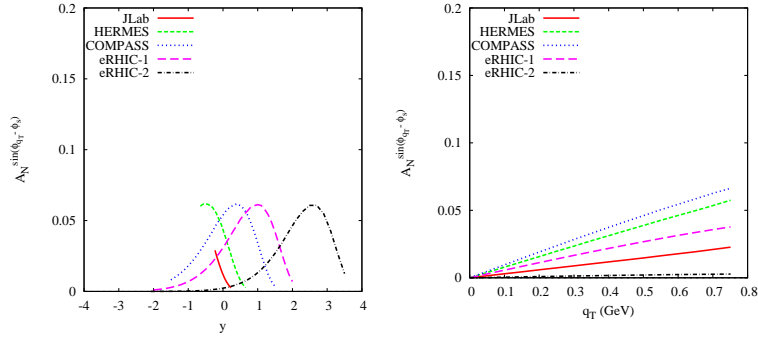
**Fig. 3** COMPASS energy ( $\sqrt{s} = 17.33 \text{ GeV}$ ), Asymmetry as a function of  $y$  (left panel) and  $q_T$  (right panel). The integration ranges are  $(0 \leq q_T \leq 1) \text{ GeV}$  and  $(-1.5 \leq y \leq 1.5)[6]$ .



**Fig. 4** eRHIC energy ( $\sqrt{s} = 31.6$  GeV), Asymmetry as a function of  $y$  (left panel) and  $q_T$  (right panel). The integration ranges are ( $0 \leq q_T \leq 1$ ) GeV and ( $-2.1 \leq y \leq 2.1$ ) [6].



**Fig. 5** eRHIC energy ( $\sqrt{s} = 158.1$  GeV), Asymmetry as a function of  $y$  (left panel) and  $q_T$  (right panel). The integration ranges are ( $0 \leq q_T \leq 1$ ) GeV and ( $-3.7 \leq y \leq 3.7$ ) [6].



**Fig. 6** Left panel: Plot of the Sivers asymmetry in the  $y$  distribution at all c.o.m energies using the TMD-e2 fit. This plot shows the drift of the asymmetry peak towards higher values of rapidity  $y$ . Right panel: Plot of the Sivers Asymmetry in the  $q_T$  distribution

## 4 Numerical Estimates

We will now present our estimates of SSA in photoproduction of  $J/\psi$  for JLAB, HERMES, COMPASS and eRHIC energies. A detailed discussion of results can be found in Ref. [6]. Figs. 1-5 show the  $y$  and  $k_T$  distribution for different experiments with parameterizations TMD Exact-1, TMD Exact -2 and TMD as given in Table 1. TMD-e1 parameter set, extracted at  $Q_0 = 1.0$  GeV, is for the exact solution of TMD evolution equations extracted in Ref. [10]. TMD-a is the parameter set fitted to analytical approximated solution of the Sivers function extracted in Ref. [10]. For estimates using NLL kernel, we have used the most recent parameters by Echevarria *et al.*[11] obtained by performing a global fit of all experimental data on Sivers asymmetry in SIDIS from HERMES, COMPASS and JLab. We call this set TMD-e2. This set was fitted at  $Q_0 = \sqrt{2.4}$  GeV. Fig. 6 shows a comparison of asymmetries at all energies.

## 5 Summary

We have compared estimates of SSA in electroproduction of  $J/\psi$  using TMD's evolved via DGLAP evolution and TMD evolution schemes. For the latter, we have chosen three different parameter sets fitted using an approximate analytical solution, an exact solution at LL and an exact solution at NLL. We find that the estimates given by TMD evolved PDF's and Sivers function are all comparable but substantially small as compared to estimates calculated using DGLAP evolved TMD's.

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